ref) Takamoto+ (2015), ApJ, 815, 16.

Relativistic Turbulent Reconnection and Application to Jet Acceleration

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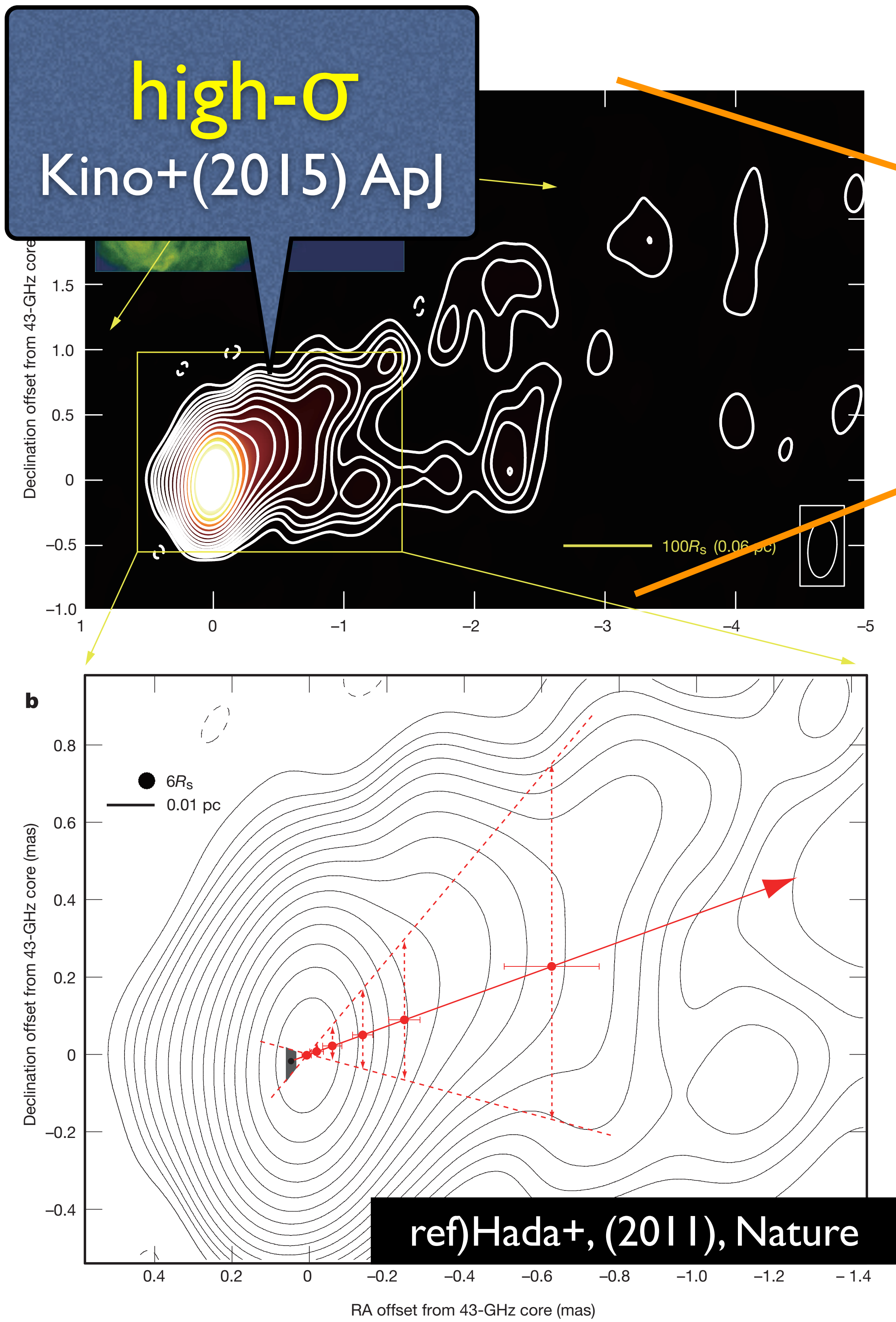
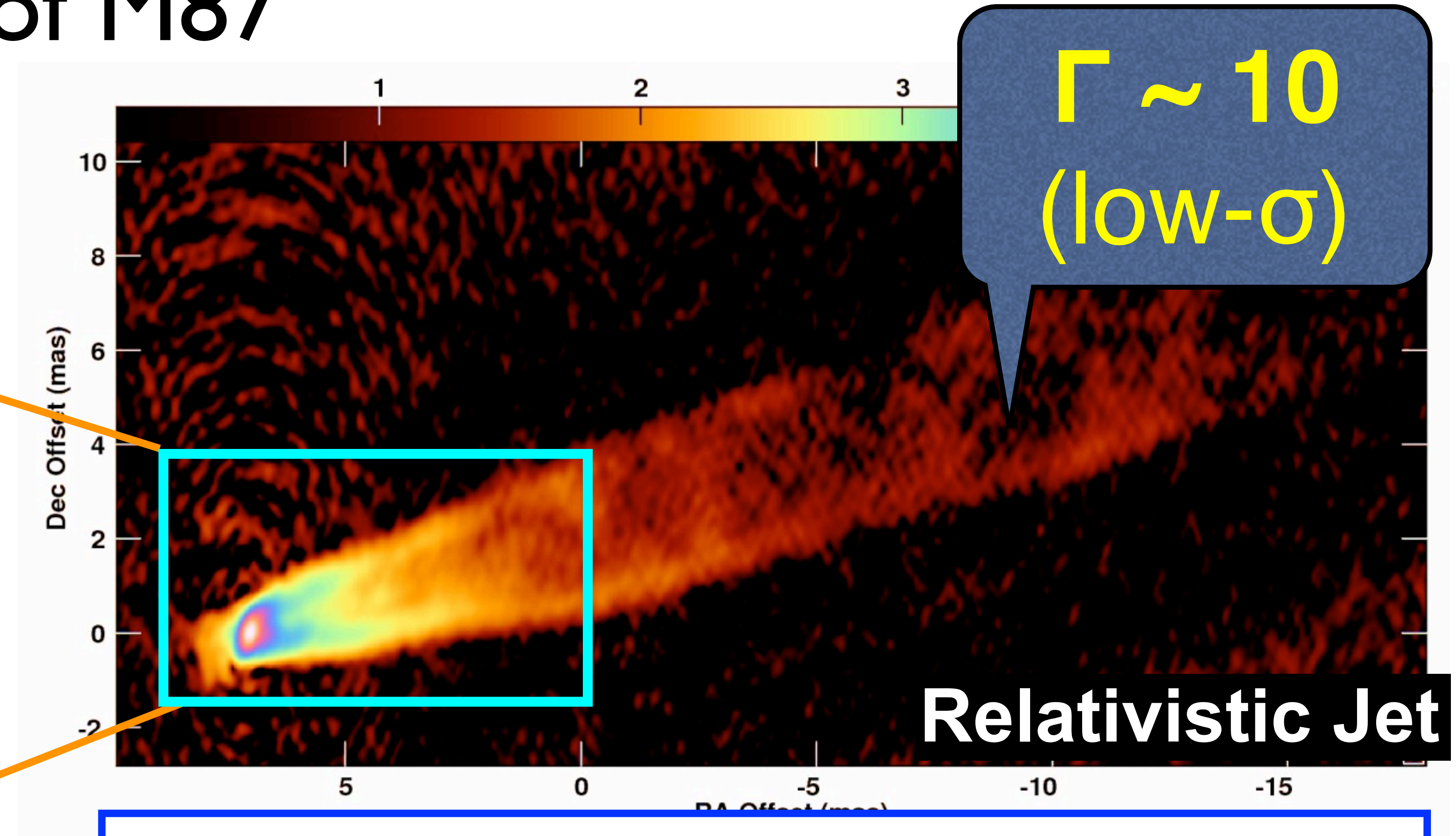
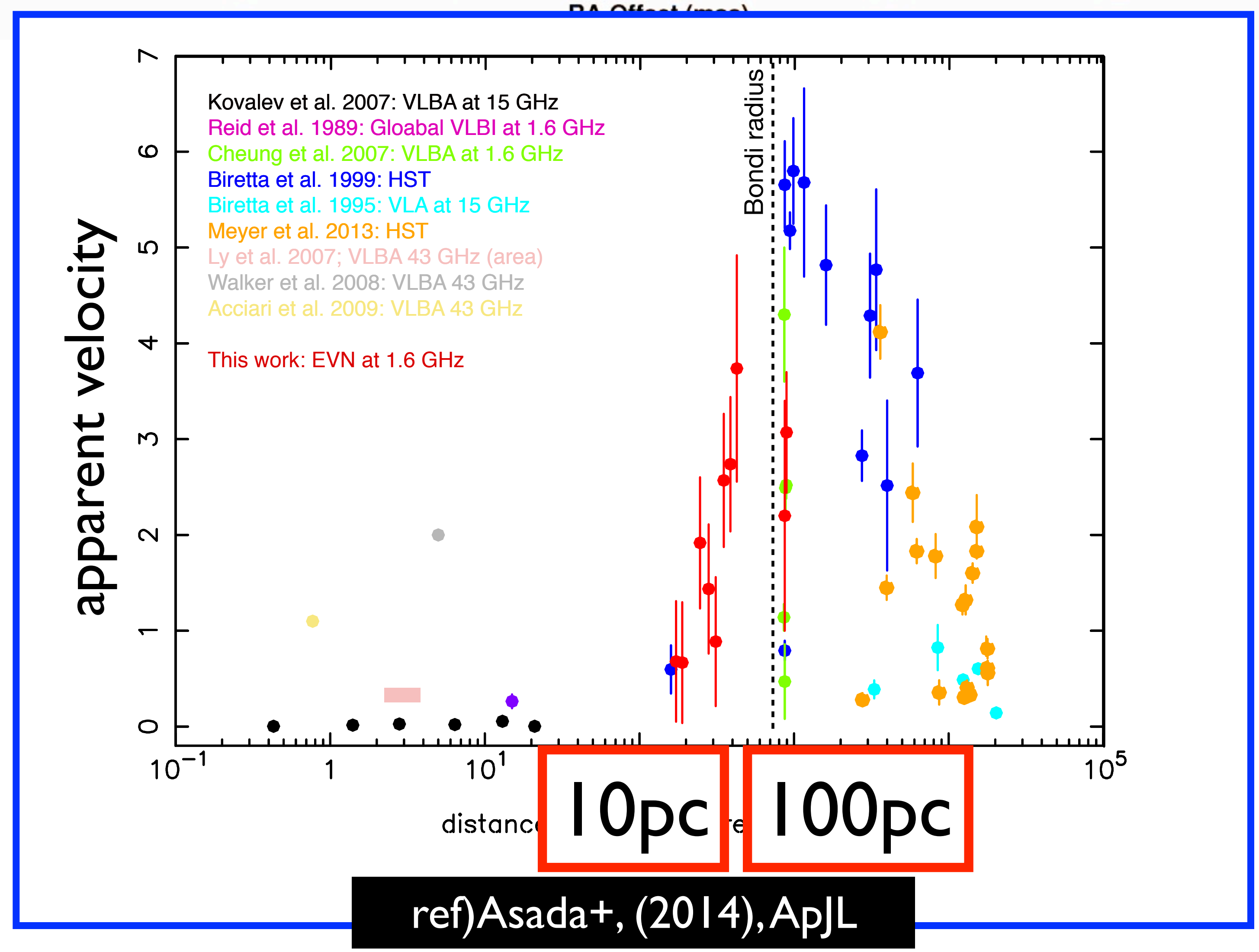
1. Recent Observations of M87







We need a very efficient magnetic field dissipation process to accelerate jets!!

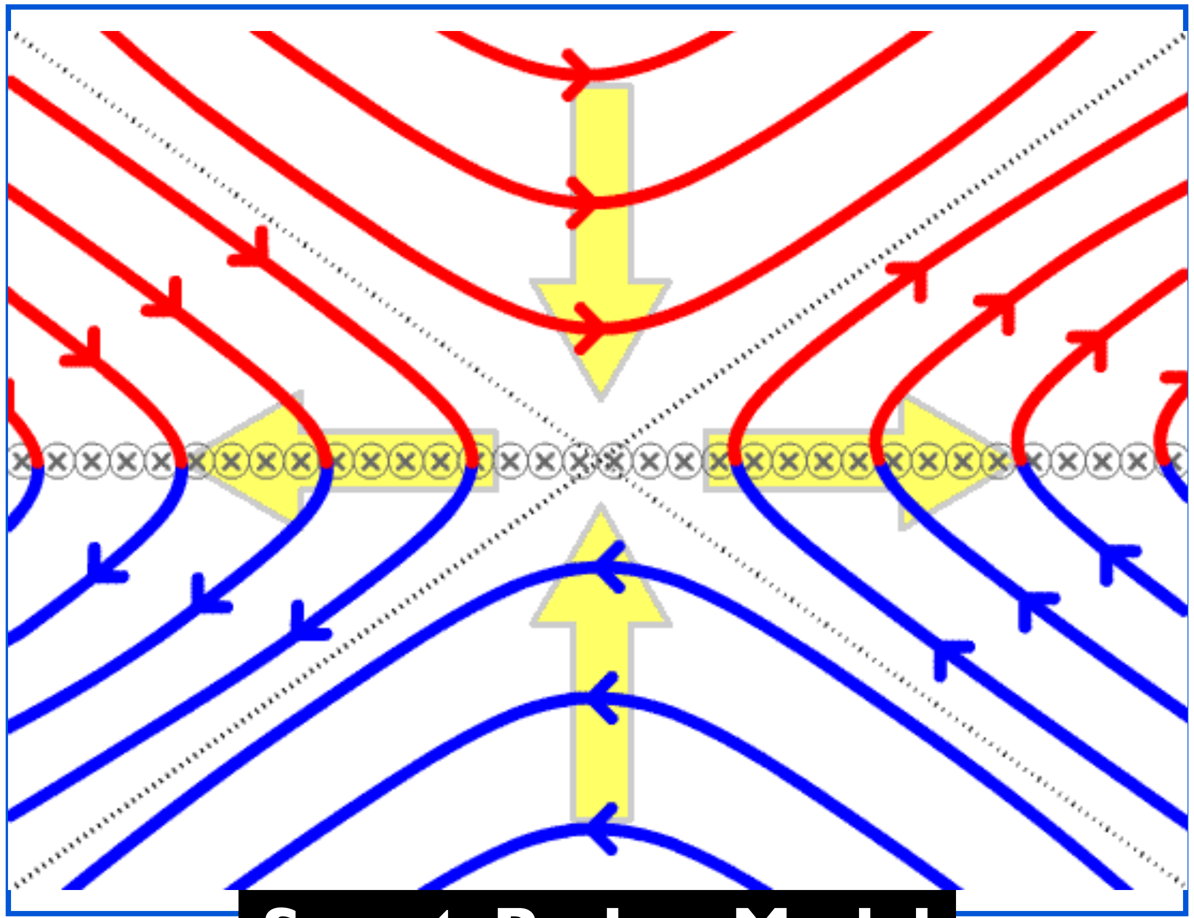
2. Magnetic Reconnection

ref) Sweet, (1958)
Parker, (1957; 1963)

Assumptions: steady flow and uniform resistivity
(Sweet-Parker model)

Reconnection rate:

$$\begin{cases} u_{in} \sim c_A / \sqrt{S} \\ S = L c_A / \eta \end{cases}$$



Sweet-Parker Model

In many astrophysical objects,

$$S = L c_A / \eta \sim L/l_{mfp} >> 1$$

$\Rightarrow$ $u_{in} \sim c_A / \sqrt{S} << c_A$

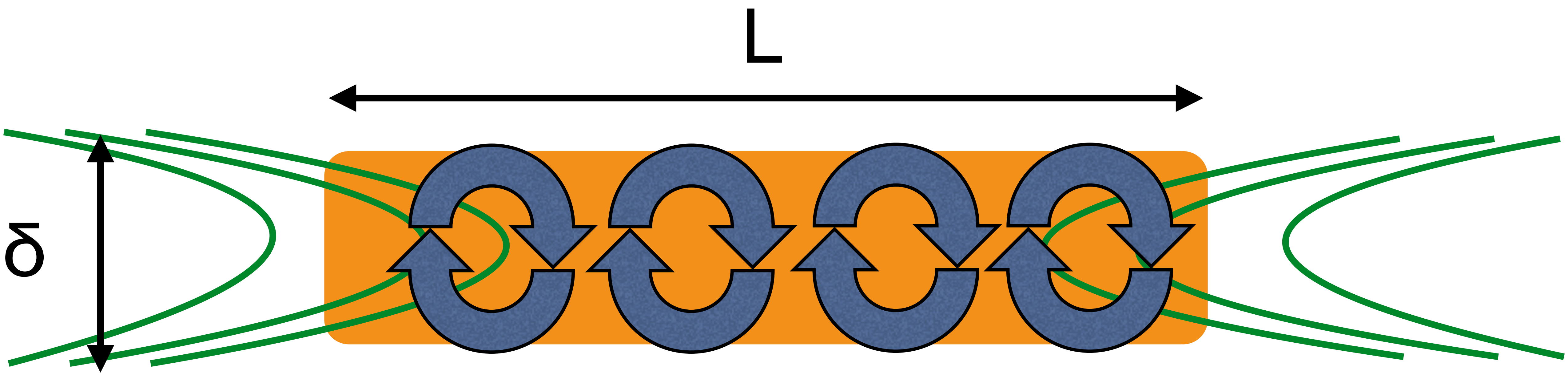
very slow ....

$\delta/L = 1/\sqrt{S}$ ⟺ too thin…

3. Turbulent Sheets

ref) Lazarian & Vishniac, (1999), ApJ, 517, 700.
Kowal et al. (2009), ApJ, 700, 63.

$$\frac{v_{\rm in}}{c_A} = \frac{\rho_{\rm out}}{\rho_{\rm in}} \frac{v_{\rm out}}{c_A} \frac{\delta}{L}$$

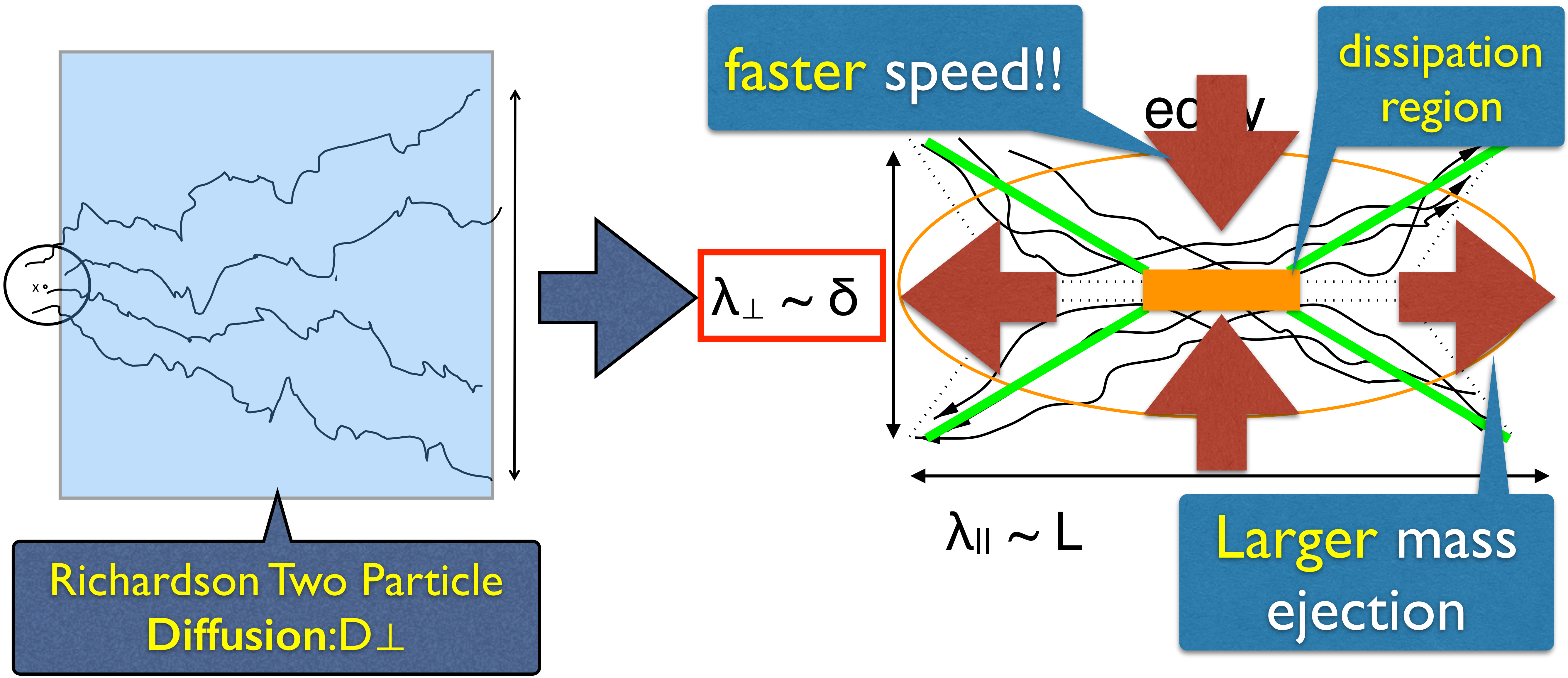


broadened by **turbulent eddies**

=> faster ! (v_R/c_A is independent of resistivity)

4. Theoretical Explanation

ref) Eyink, Lazarian, Vishniac, (2011), ApJ, 743, 51.



$$\frac{\delta}{L_x} = M_A^2 \min\left\{ \left(\frac{L_x}{L_i}\right)^{1/2}, \left(\frac{L_i}{L_x}\right)^{1/2} \right\}$$

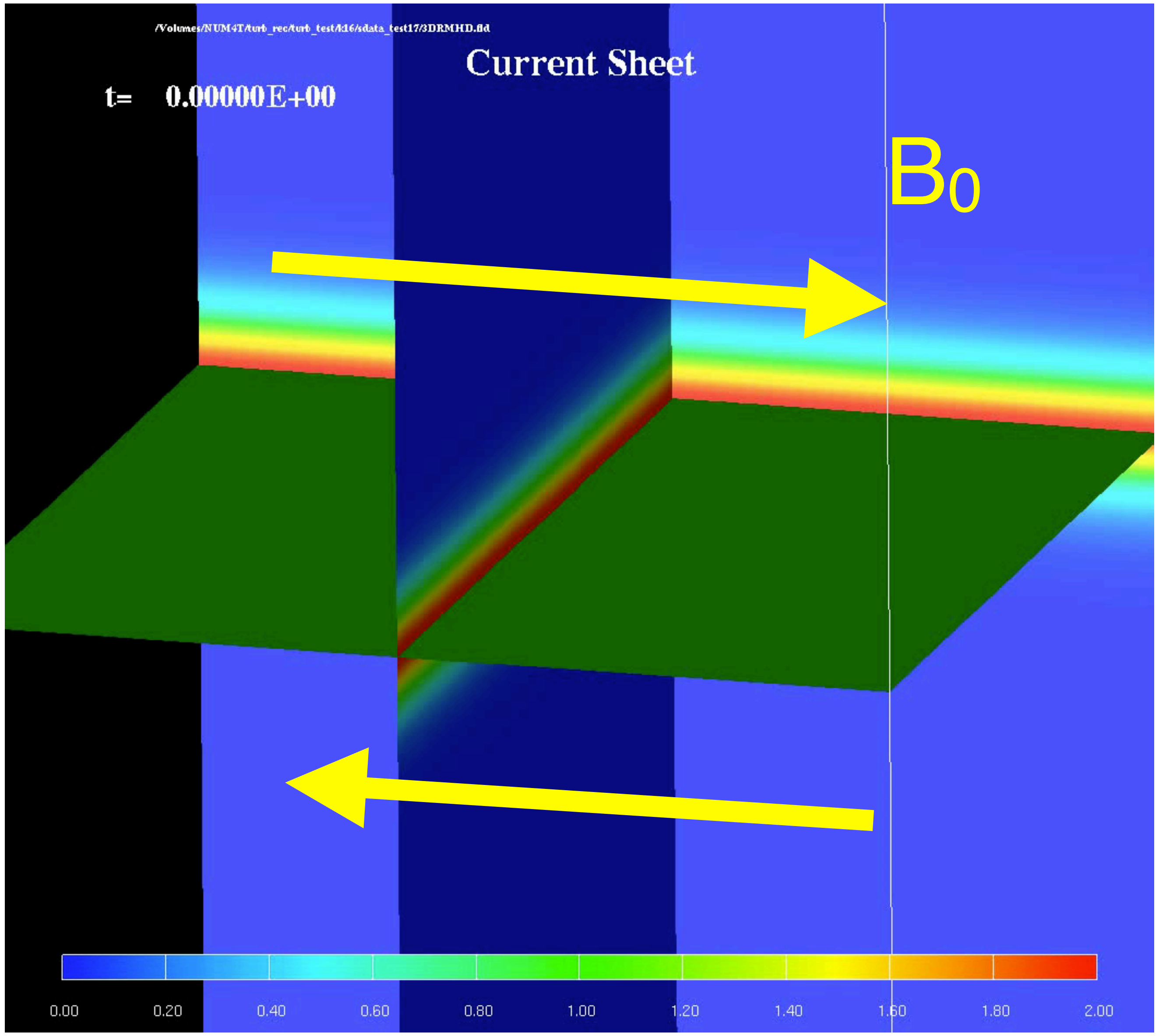
What happens

in relativistic cases??

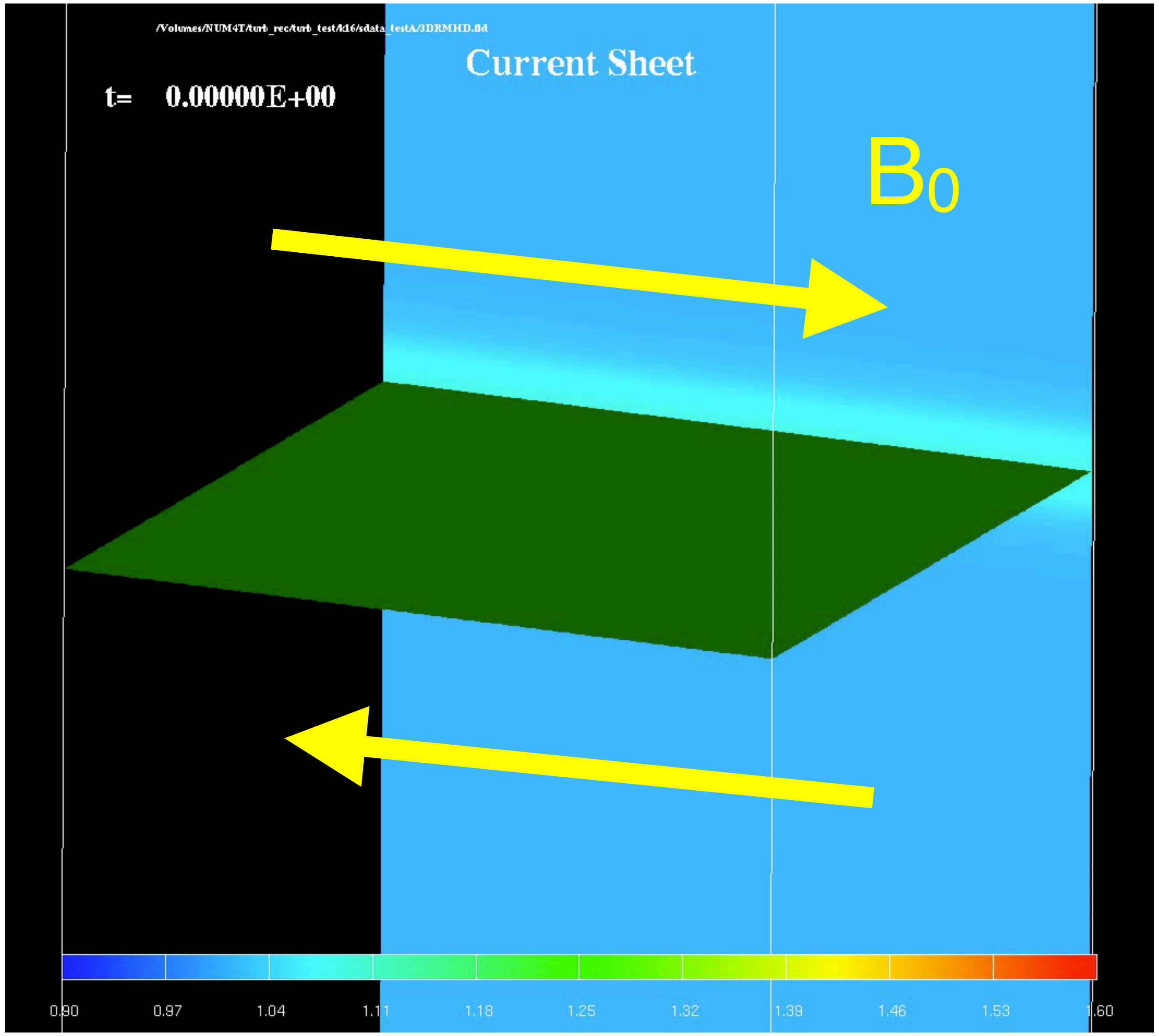
5. Relativistic Turbulent Reconnection

ref) Takamoto+ (2015), ApJ, 815, 16.

Poynting Dominated($\sigma = 5$)

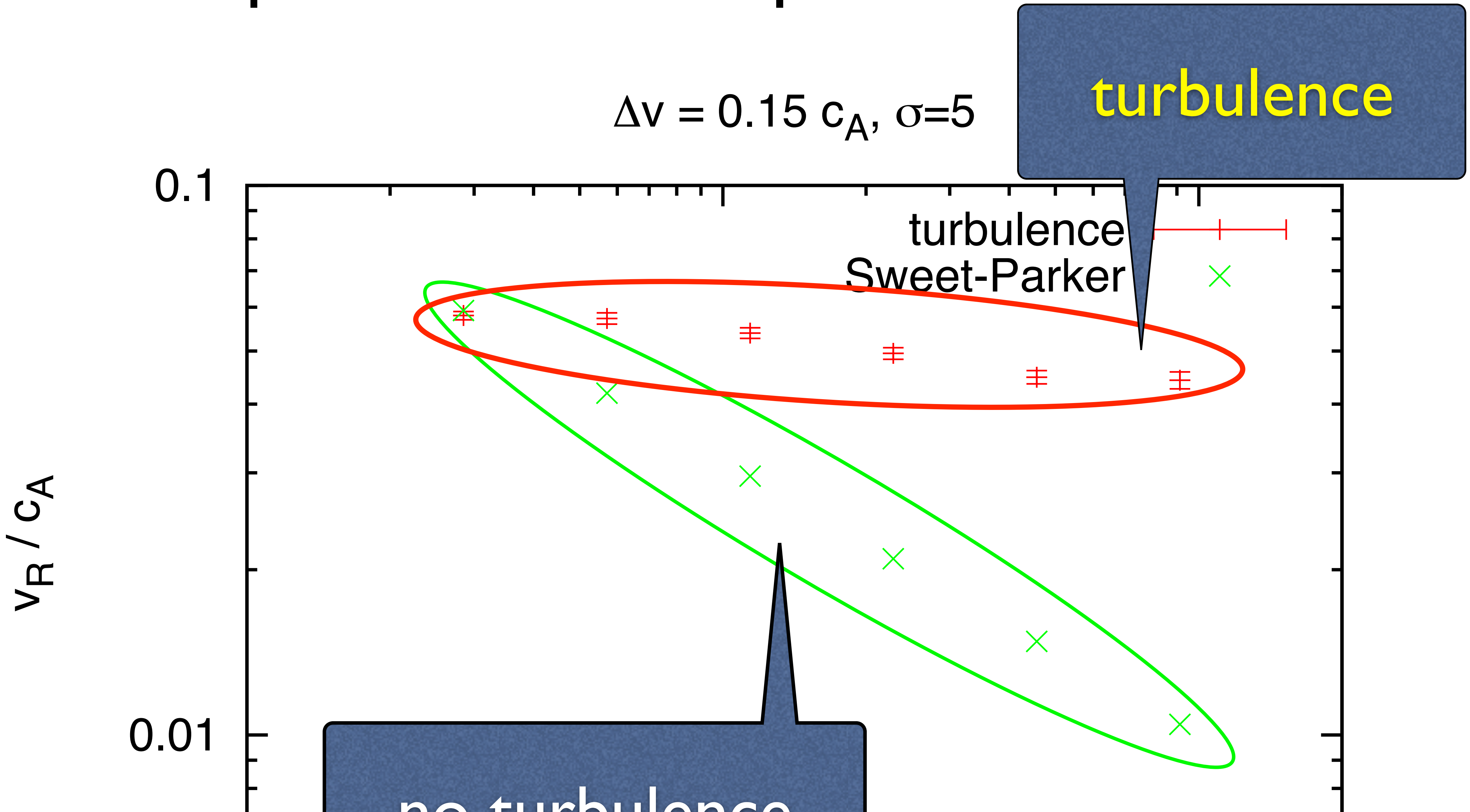


Matter dominated($\sigma = 0.04$)



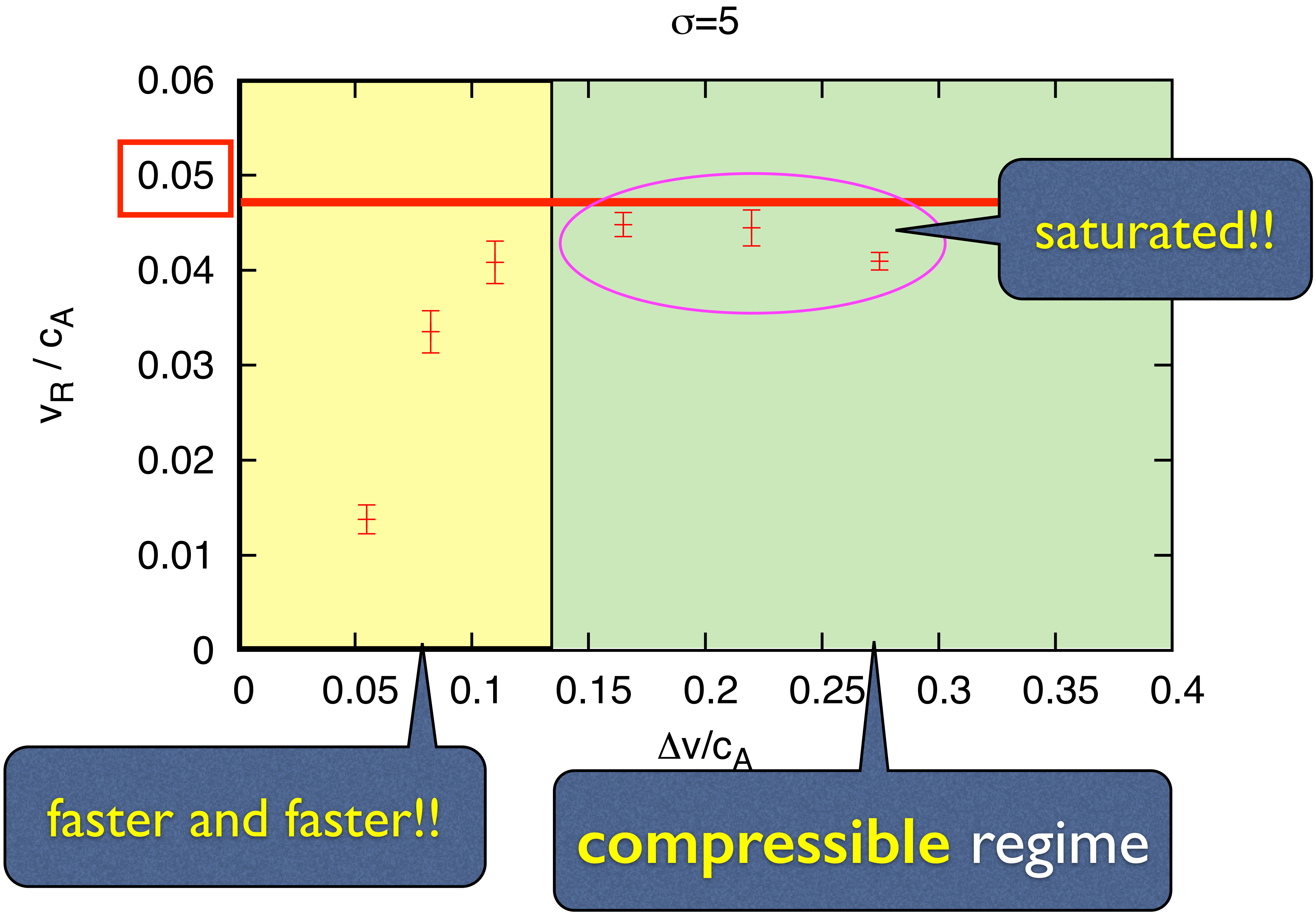
- $k_B T/mc^2 = 1$
- **driven** turbulence
  injected around central region

6. Lundquist Number Dependence



fast & resistivity independent
mechanism

7. Turbulence-Strength Dependence



8. Necessary Turbulence Energy in Jets

if we set:

$$\frac{v_{\rm turb}}{c_{\rm A}} \equiv \alpha$$

3-Mach Number

$$\frac{\epsilon_{\rm turb}}{\epsilon_{\rm B}} \equiv \frac{\rho_0 h v_{\rm turb}^2/2}{B_0^2/8\pi} = \frac{\alpha^2}{1+\sigma}$$

$$\left(c_{\rm A} \equiv c\sqrt{\frac{\sigma}{1+\sigma}}, \quad \sigma \equiv \frac{B^2}{4\pi\rho_0 h c^2\gamma^2} = \frac{B_0^2}{4\pi\rho_0 h c^2} \right)$$

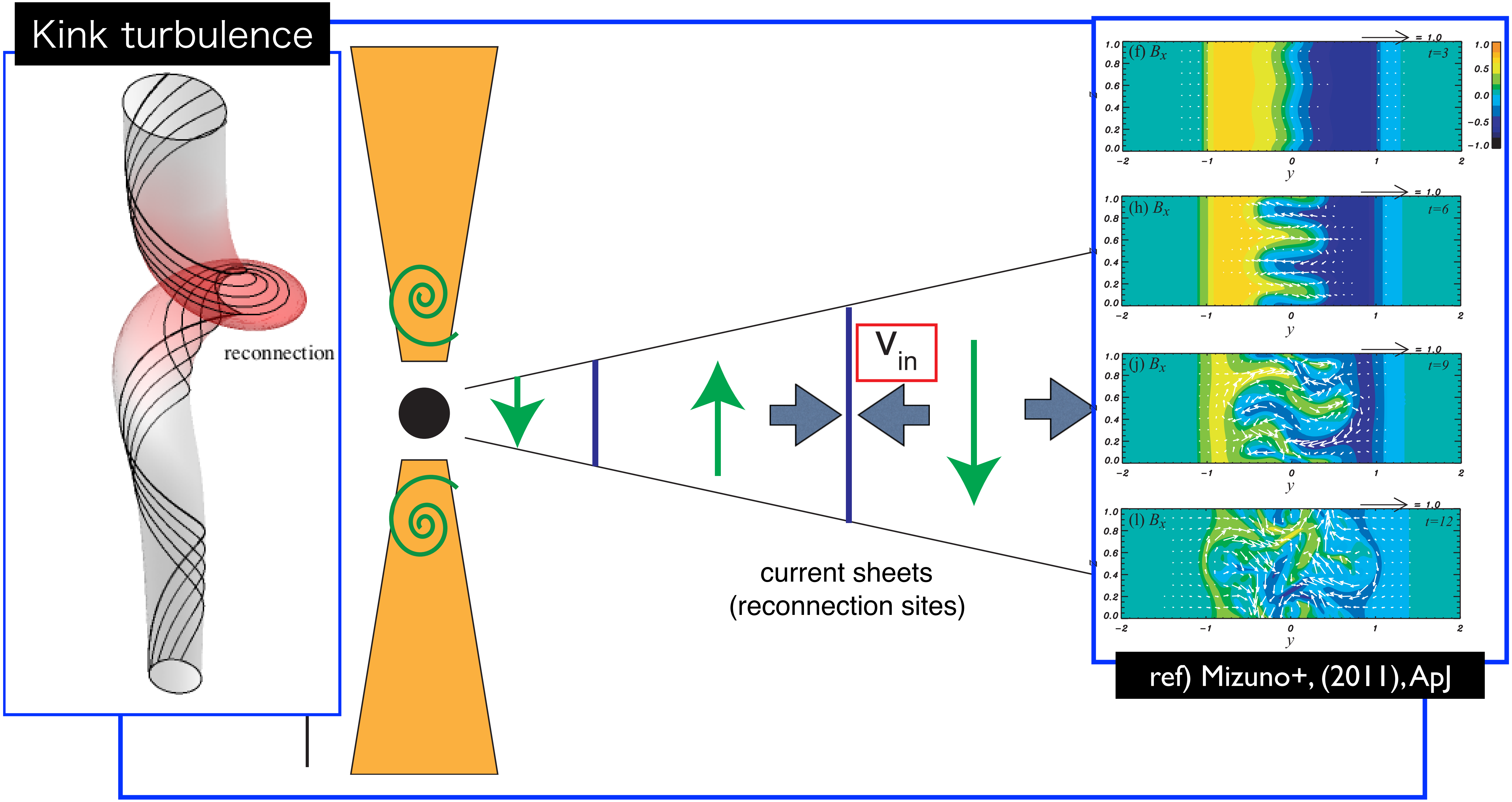
if we assume: $\alpha = 0.3$, $\sigma = 10$,

$\epsilon_{\rm turb} / \epsilon_{\rm B} \sim 0.01$

just **1%** of magnetic field energy is sufficient!!

9. Application — Relativistic Jets

ref) MT+,(2015), ApJ, 812, 15
Rieger & Aharonian, (2012), MPLA 27, 30030



$$\frac{v_{\rm in}}{c} \gtrsim 1.9 \times 10^{-3} \left(\frac{l_{\rm jet}}{60[{\rm pc}]}\right)^{-1} \left(\frac{r_{\rm MRI}}{3 r_M}\right)^{3/2} \left(\frac{r_M}{10^{-4}[{\rm pc}]}\right)^{-1/2} \left(\frac{\Gamma_{\rm jet}}{5}\right)^{2}$$

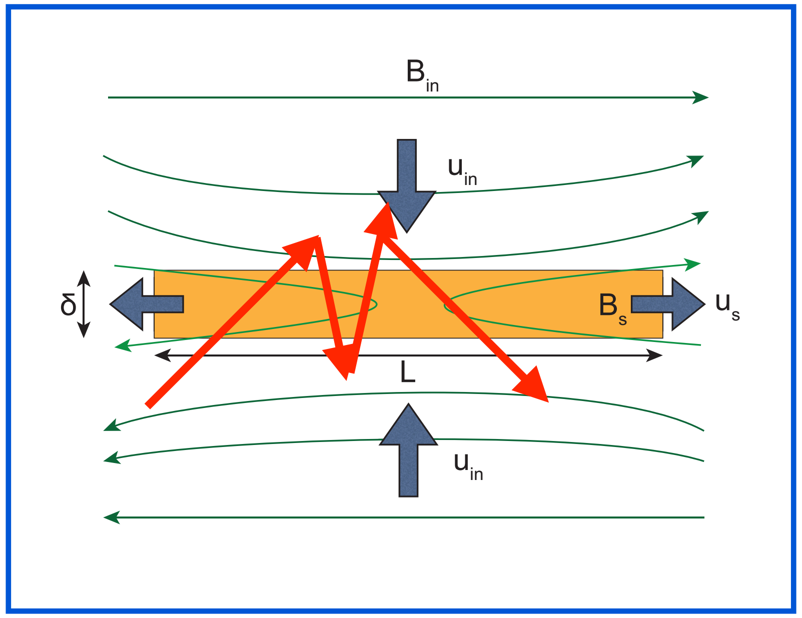
10. Particle Acceleration by Reconnection

- If Turbulent Reconnection:

→ shock-like acceleration

ref) Pino&Lazarian, (2005), A&A 441, 845.

$$N(E) \propto E^{-2.5}$$



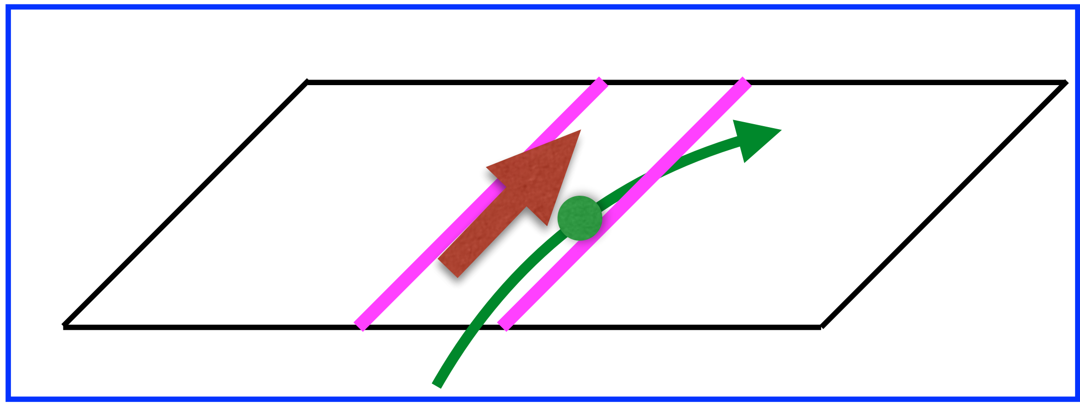
- X-point Acceleration:

→ direct acceleration

by electric field at X-point

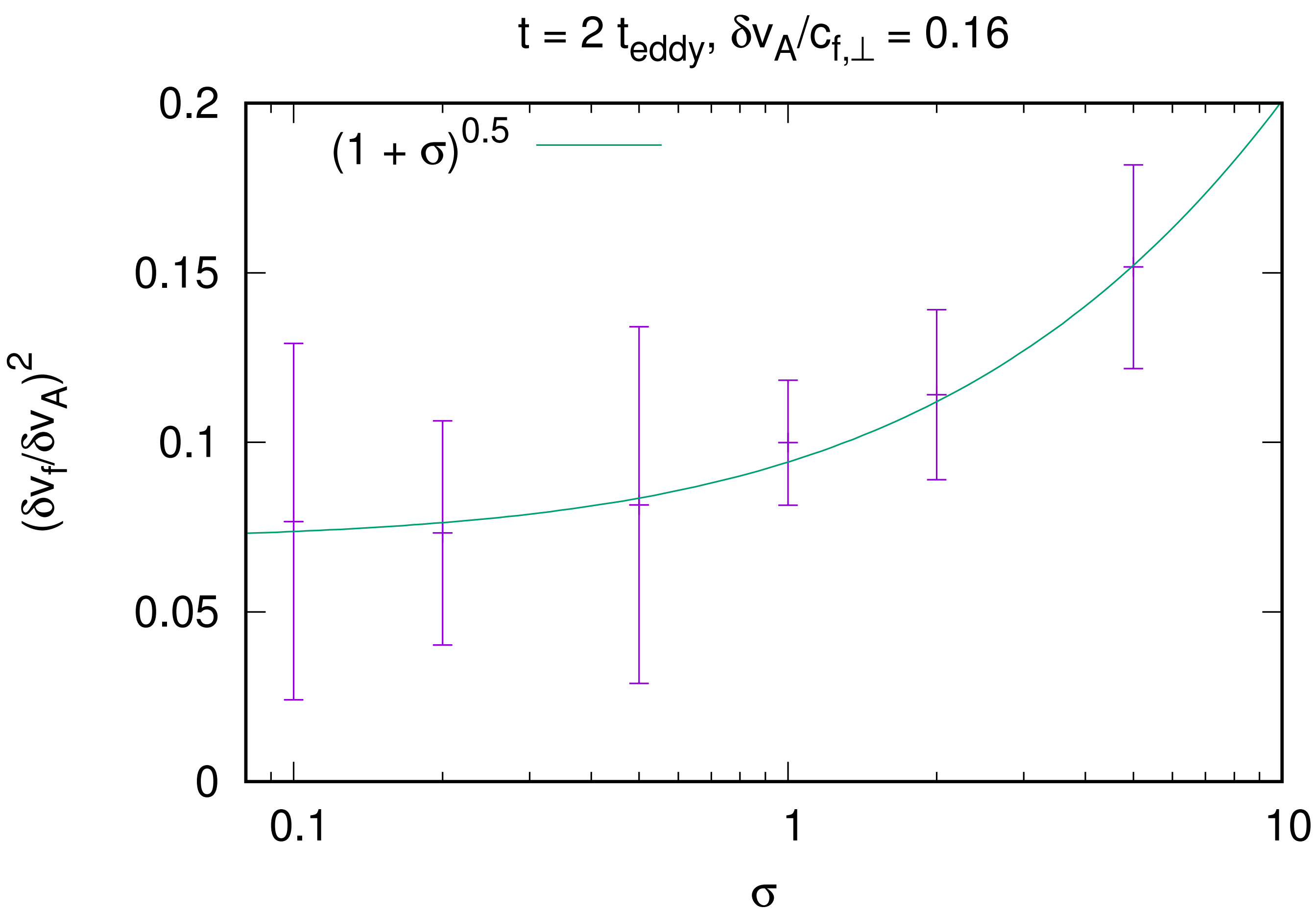
ref) Zenitani & Hoshino (2001), ApJL, 562, 63.
Bessho & Bhattacharjee (2012), ApJ, 750, 129.
Sironi & Spitkovsky, (2014), ApJL, 783, 21.

$$N(E) \propto E^{-1.4}$$



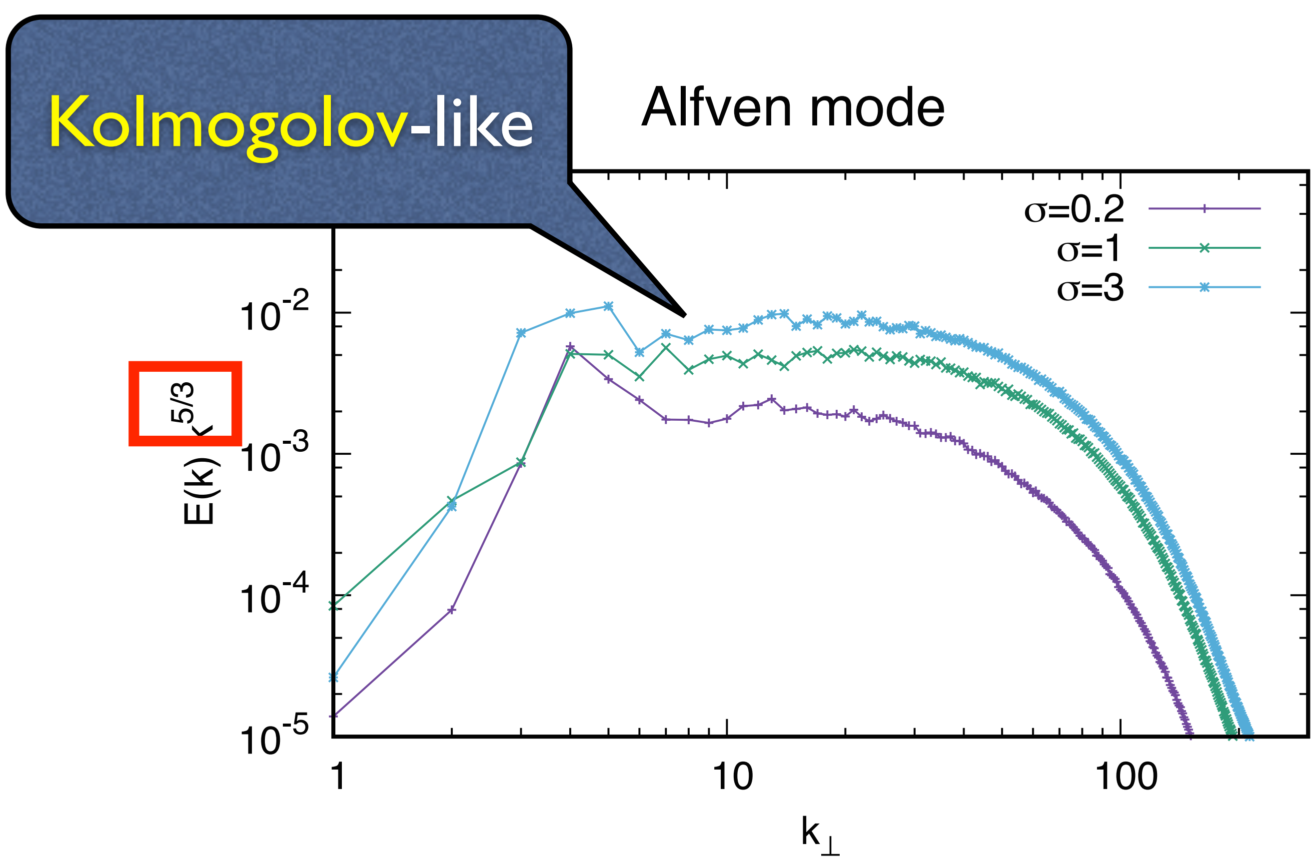
11. Relativistic MHD Turbulence

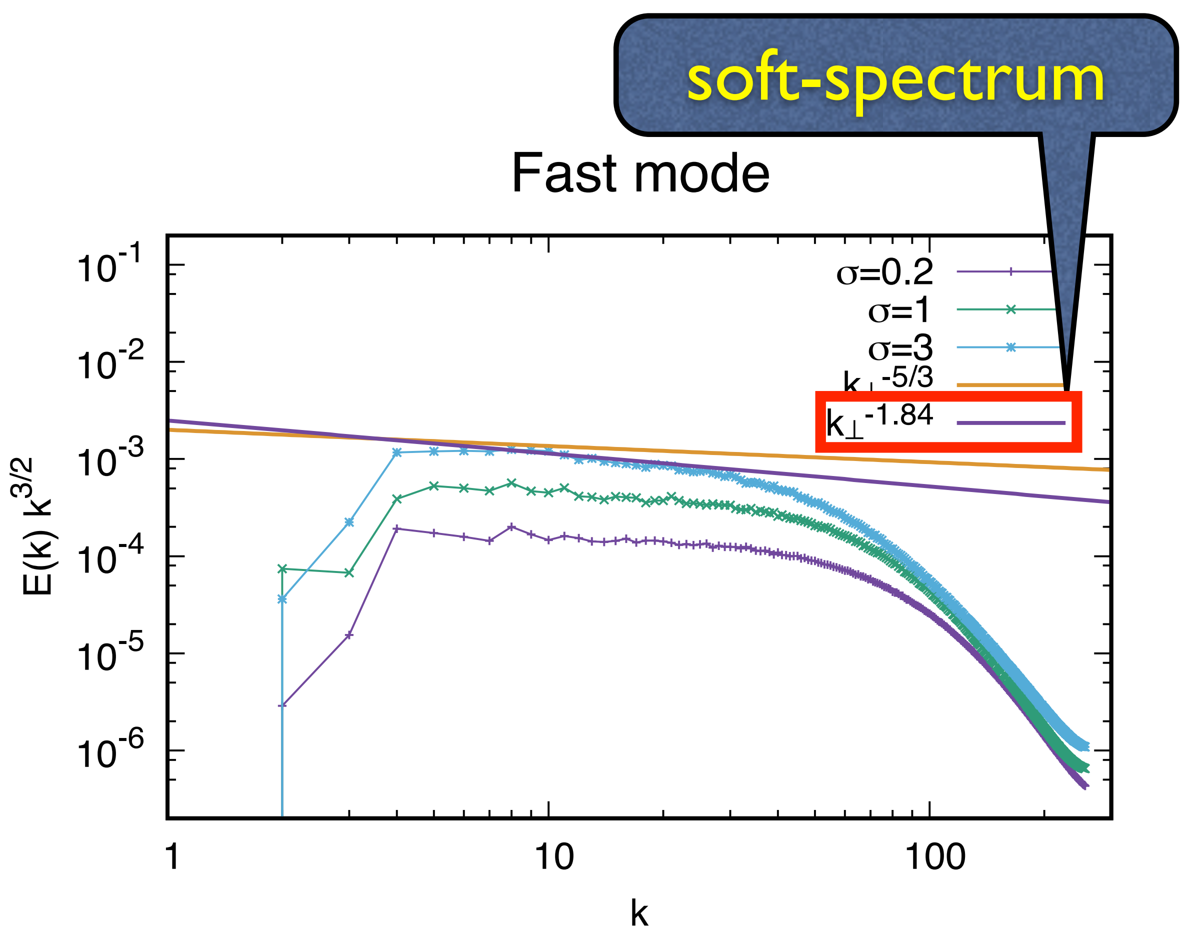
ref) MT & Lazarian (2016), submitted to PRL



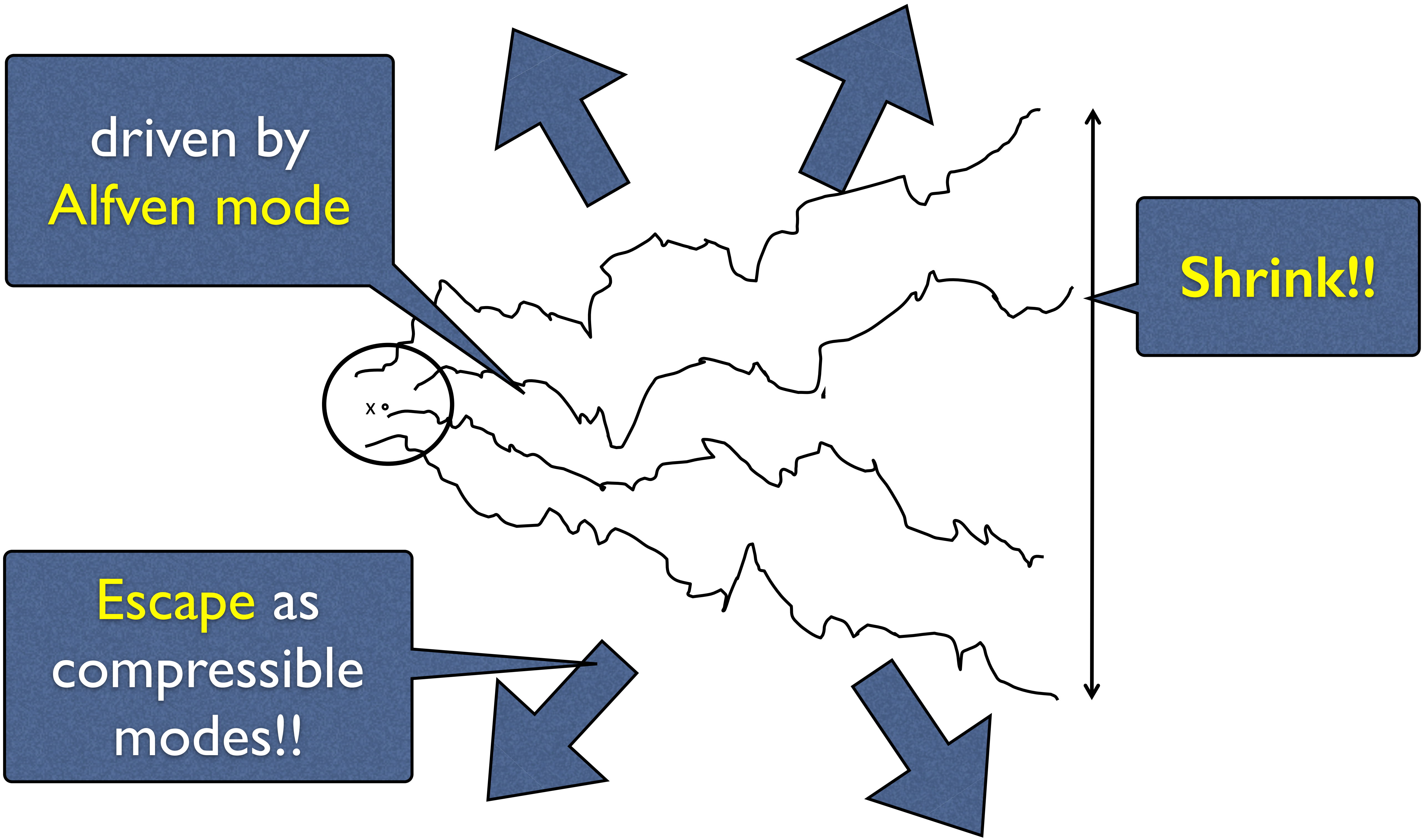
At a fixed Alfvénic velocity,

$$(\delta V)_{\rm f}^2/(\delta V)_{\rm A}^2 \propto (1+\sigma)^{1/2}(\delta V)_{\rm A}/c_{{\rm fast},\perp}$$





12. Compressible Effect



13. Compressible Effects

$$\frac{v_{\rm in}}{c_A} = \frac{\rho_{\rm out}}{\rho_{\rm in}} \frac{v_{\rm out}}{c_A} \frac{\delta}{L}$$

Incompressible: (LV99)

$$\frac{\delta}{L} \simeq \min\left[\left(\frac{L}{l}\right)^{1/2}, \left(\frac{l}{L}\right)^{1/2}\right] \left(\frac{v_l}{c_A}\right)^2$$

escaping as compressible modes

compressible:

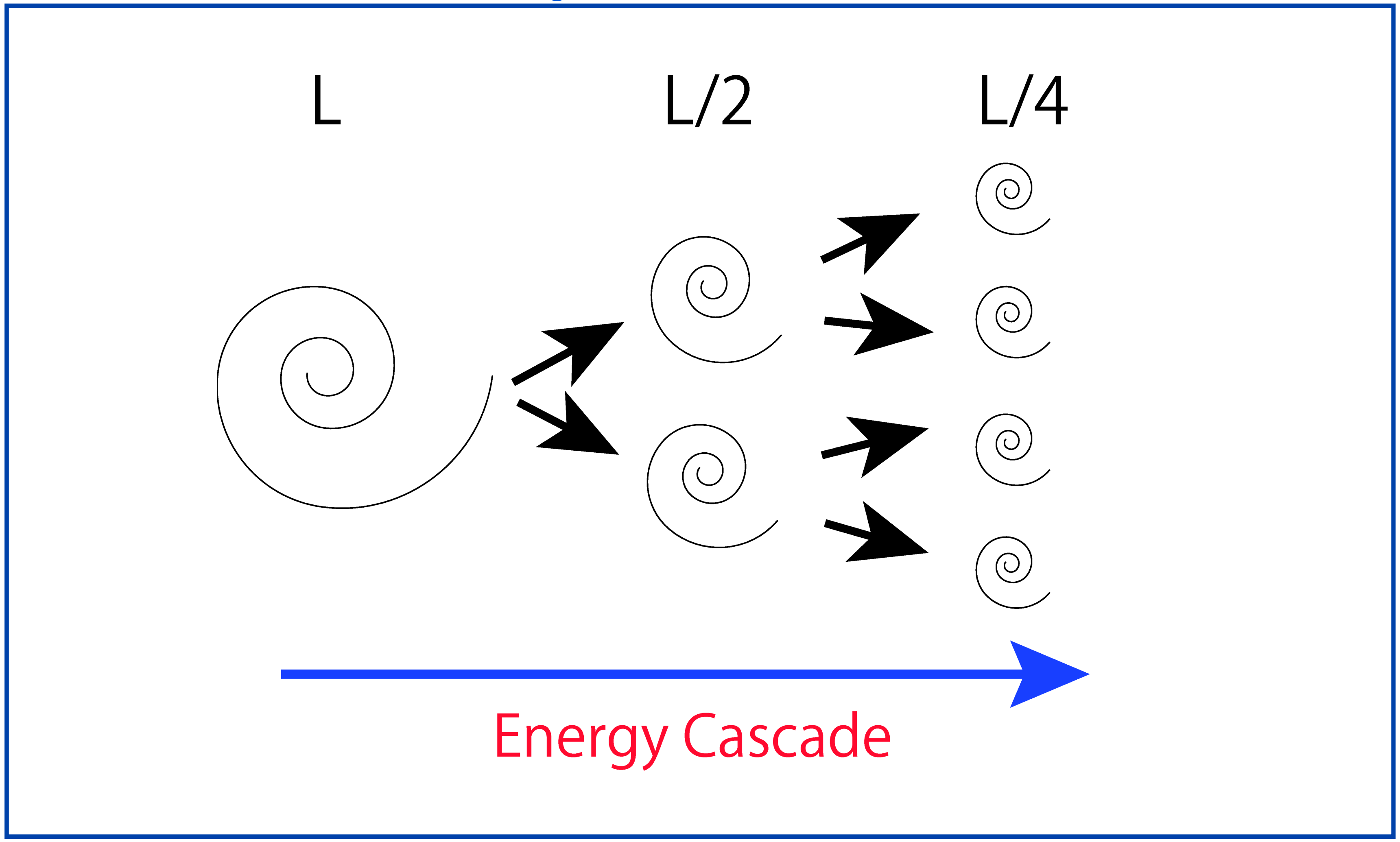
$$\frac{\delta}{L} \simeq \min\left[\left(\frac{L}{l}\right)^{1/2}, \left(\frac{l}{L}\right)^{1/2}\right] \left[\left(\frac{v_l}{c_A}\right)^2 - C\left(\frac{v_l}{c_A}\right)^4\right]$$

O Ma Ke

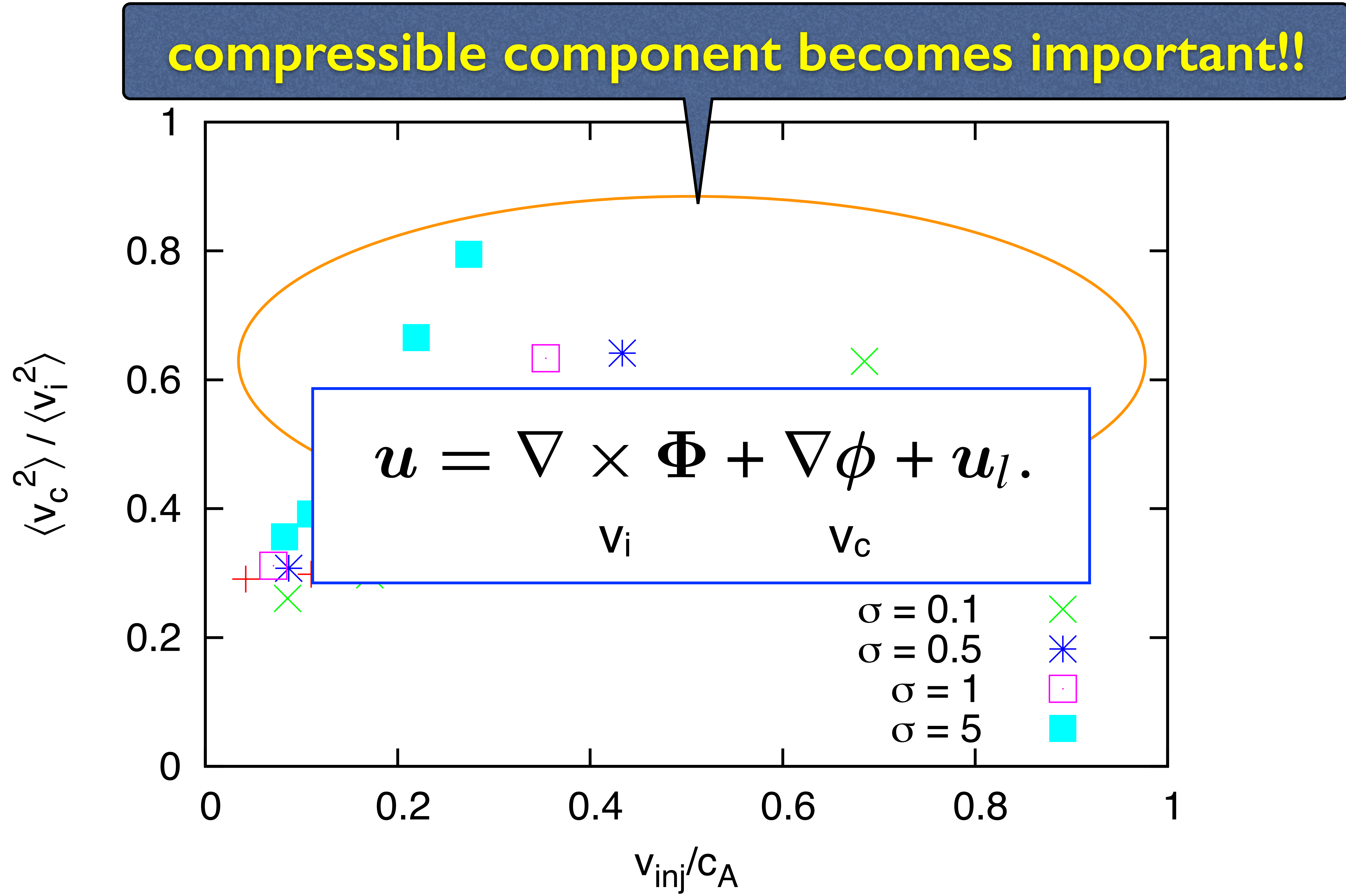
Kolmogorov Turbulence

Assumptions: · Homogeneous and isotropic turbulence

· Steady state



$$\left\{ \begin{array}{l} v_l = v_L (l/L)^{1/3} \propto l^{1/3} \text{ :eddy velocity} \\ E(k) \propto k^{-5/3} \text{ : Energy Spectrum} \end{array} \right.$$


compressible component becomes important!!
$$\boldsymbol{u} = \nabla \times \boldsymbol{\Phi} + \nabla\phi + \boldsymbol{u}_l.$$
v_i
v_c
$\langle v_c^2\rangle / \langle v_i^2\rangle$
v_{inj}/c_A
σ = 0.1
σ = 0.5
σ = 1
σ = 5
0
0.2
0.4
0.6
0.8
1

MHD Turbulence (Goldreich-Sridhar model)

Assumptions:
- Magnetic Field exists
- Steady state

Features:
- eddy is enlarged along B

$$k_{||} \propto k_{\perp}^{2/3}$$

- Turbulent motion perpendicular to B obeys Kolmogolov law

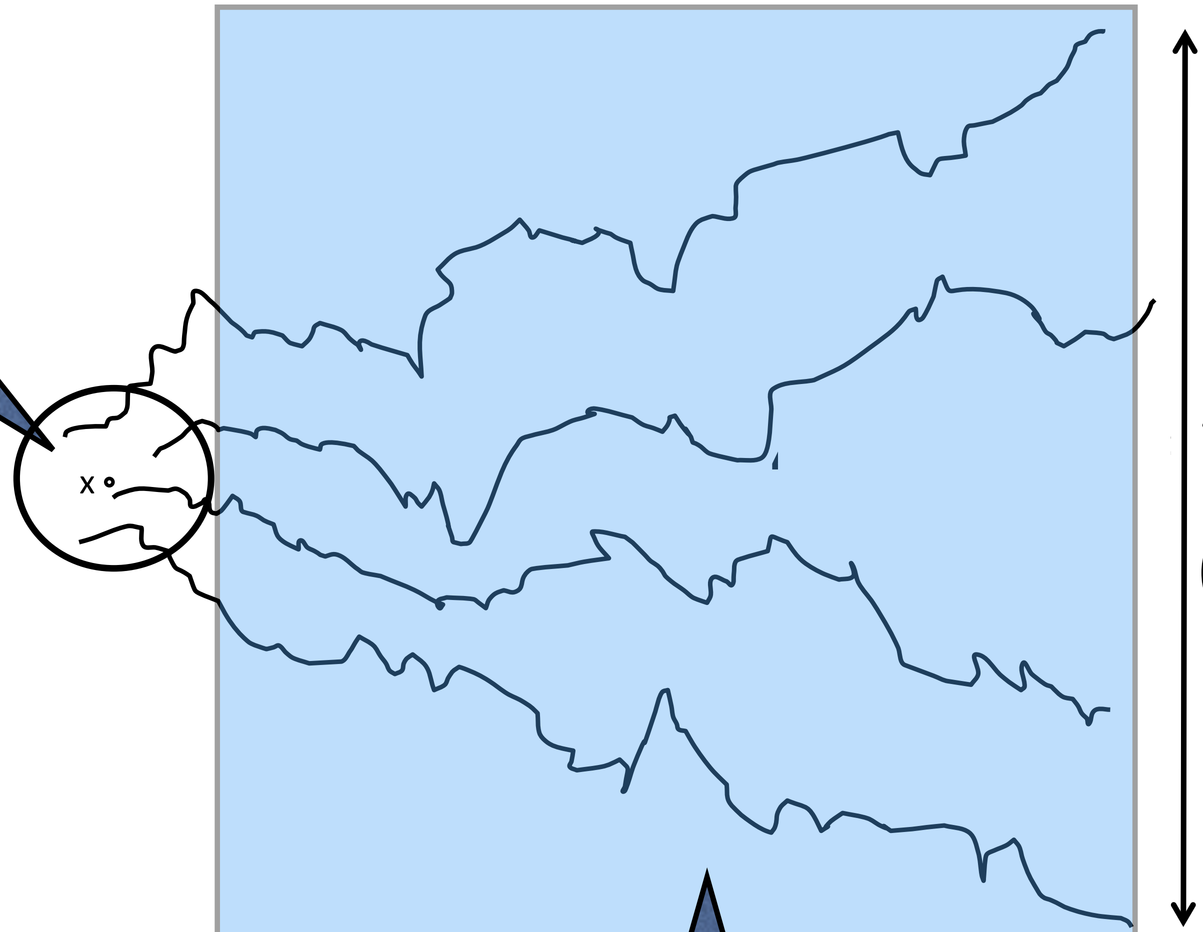
$$E(k_{\perp}) \propto k_{\perp}^{-5/3}$$

$$k_{||} c_A \sim k_{\perp} v_k$$:critical balance

3.1. Theoretical Explanation

ref) Eyink, Lazarian, Vishniac, (2011), ApJ, 743, 51.

initial location



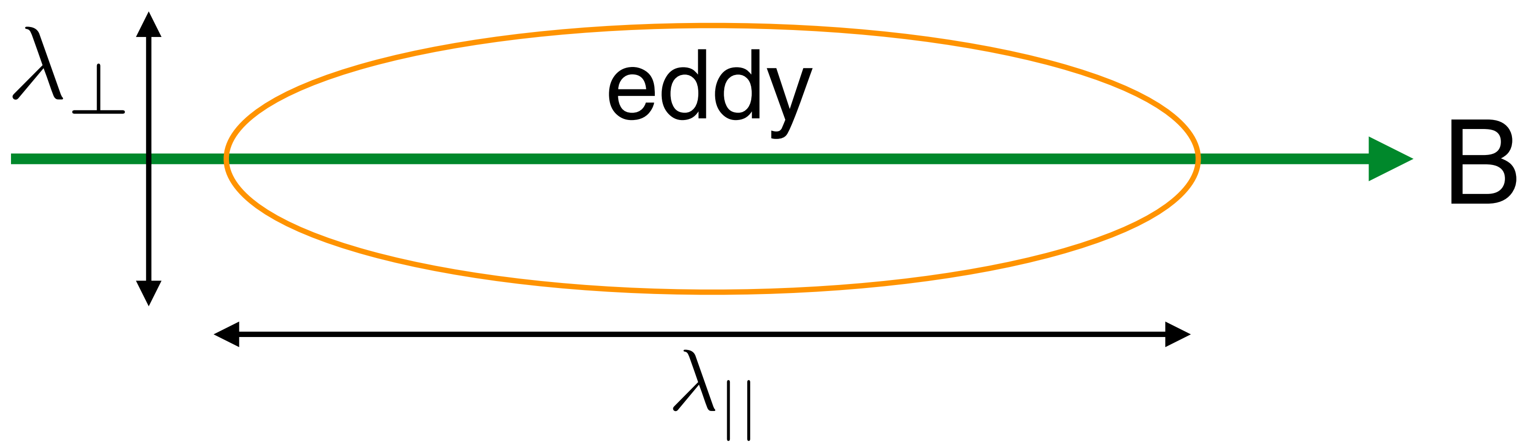
$\langle y^2(t) \rangle \sim (L_x^3/L_i) M_A^4 (t/t_A)^3$

Diffusion= independent of initial width

Richardson Two Particle Diffusion:D⊥

$$\frac{\delta}{L_x} = M_A^2 \min\left\{ \left(\frac{L_x}{L_i}\right)^{1/2}, \left(\frac{L_i}{L_x}\right)^{1/2} \right\}$$

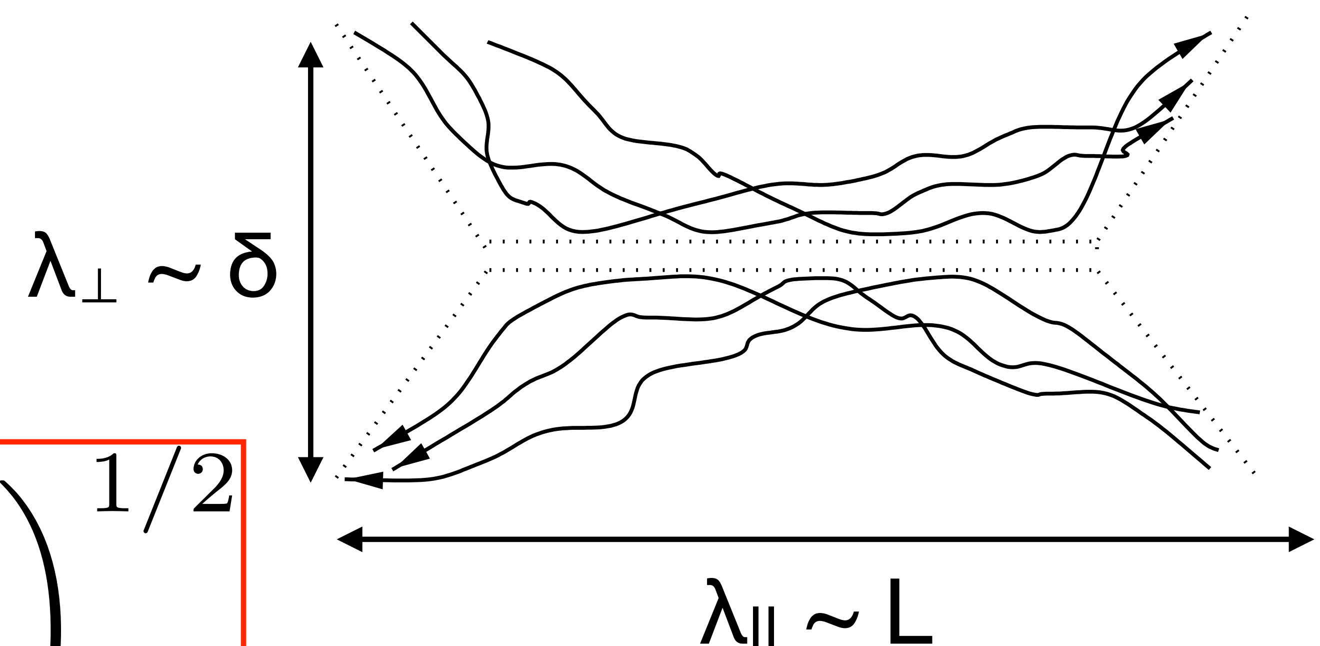
3.1. Theoretical Explanation



$$\frac{\lambda_{||}}{l} \sim \left(\frac{\lambda_\perp}{l}\right)^{2/3} \left(\frac{v_l}{c_A}\right)^{-4/3}$$: MHD turbulence

$$\frac{v_{\rm in}}{c_A} = \frac{\delta}{L} \sim \frac{\lambda_\perp}{\lambda_{||}}$$

$$\sim \left(\frac{v_l}{c_A}\right)^{2} \left(\frac{l}{L}\right)^{1/2}$$



Relativistic Ideal Fluid

Basic equations of relativistic hydrodynamics (RHD):

$$D\rho = -\rho\nabla_\mu u^\mu \quad \text{:Mass Conservation}$$

$$\rho h D u_\mu = -\nabla_\mu p \quad \text{: Equation of Motion}$$

$$\rho D e = -p\nabla_\mu u^\mu \quad \text{: Equation of Energy}$$

$$\gamma_{\mu\nu} = \eta_{\mu\nu} + u_\mu u_\nu \quad \text{: spatial projection tensor}$$

$$\partial_\mu = \eta_{\mu\nu}\partial^\nu = (-u_\mu u_\nu + \gamma_{\mu\nu})\partial^\nu$$

$$\equiv -u_\mu D + \nabla_\mu \quad \text{:3+1 decomposition}$$

2.5. Relativistic Magnetohydrodynamics

Basic equations of RMHD:

$$
\begin{cases}
\partial_t(\rho\gamma) + \partial_i(\rho\gamma v^i) = 0, \\
\partial_t(\rho h_{tot}\gamma^2 v^j - b^0 b^j) + \partial_i(\rho h_{tot}\gamma^2 v^i v^j + p_{tot}\delta^{ij} - b^i b^j) = 0, \\
\partial_t(\rho h_{tot}\gamma^2 - p_{tot} - (b^0)^2) + \partial_i(\rho h_{tot}\gamma^2 v^i - b^0 b^i) = 0, \\
\partial_t B^j + \partial_i(v^i B^j - B^i v^j) = 0, \quad \partial_i B^i = 0. \\
\qquad h_{tot} = 1 + \epsilon + \frac{b^2}{\rho}, \quad p_{tot} = p_{gas} + \frac{b^2}{2}
\end{cases}
$$

features:
- correction from Lorentz factor and inertia of energy
- tension and pressure from magnetic field

2.3. Relativistic Effects:

Lorentz contraction:

lab frame density: $\rho \Rightarrow \rho\gamma$

larger density

Alfven velocity:

$$c_A/c = \frac{B}{\sqrt{4\pi\rho h + B^2}} < 1$$

sub-luminal

Electric Field:

$$qE \sim (\nabla E)E \sim v^2 B^2/R \sim p_B \mathbf{v}\cdot\nabla\mathbf{v}$$

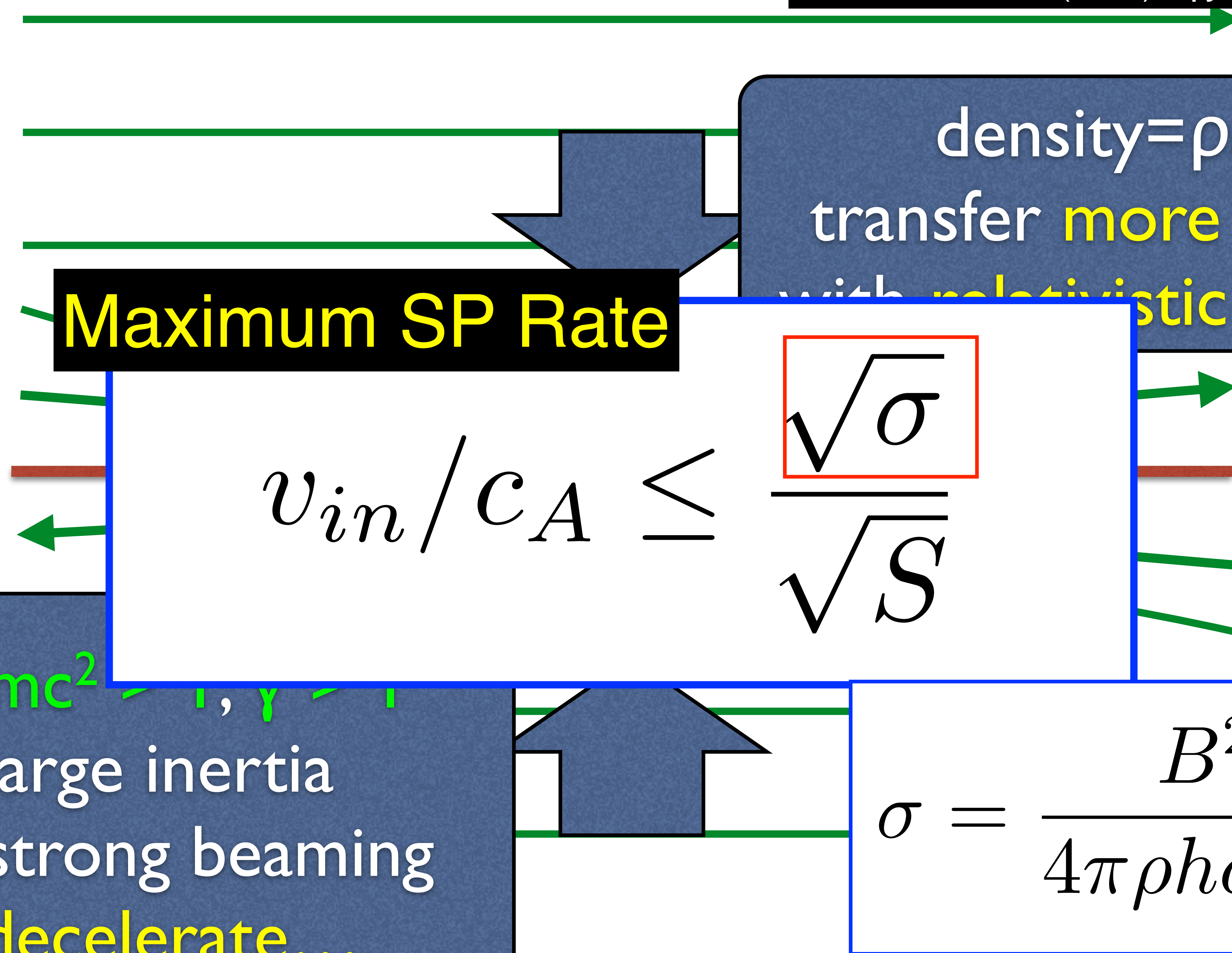
$$j \times B \sim (\nabla \times B - \partial_t E) \times B \sim (\nabla \times B - \partial_t v B) \times B$$

$$\sim (\nabla \times B) \times B - p_B \partial_t v$$

inertia from magnetic field

2.4. Relativistic Effects on Reconnection

ref) Lyutikov&Uzdensky 2003, ApJ 589, 893
Lyubarsky, (2005), ApJ, 358, 113.
Zenitani etal, (2009), ApJ 696, 1385.



density= $\rho_0 \gamma$
transfer more matter
with relativistic velocity

Maximum SP Rate

$$v_{in}/c_A \leq \frac{\sqrt{\sigma}}{\sqrt{S}}$$

$k_B T/mc^2 > 1$, $\gamma > 1$
= large inertia
strong beaming
=>decelerate…
thin sheet…

$$\sigma = \frac{B^2}{4\pi\rho h c^2 \gamma^2}$$

3.6. Compressible Effect: 2

ref) Banerjee & Galtier, PRE 87, 013019, (2013).

Energy cascade law in MHD turbulence:

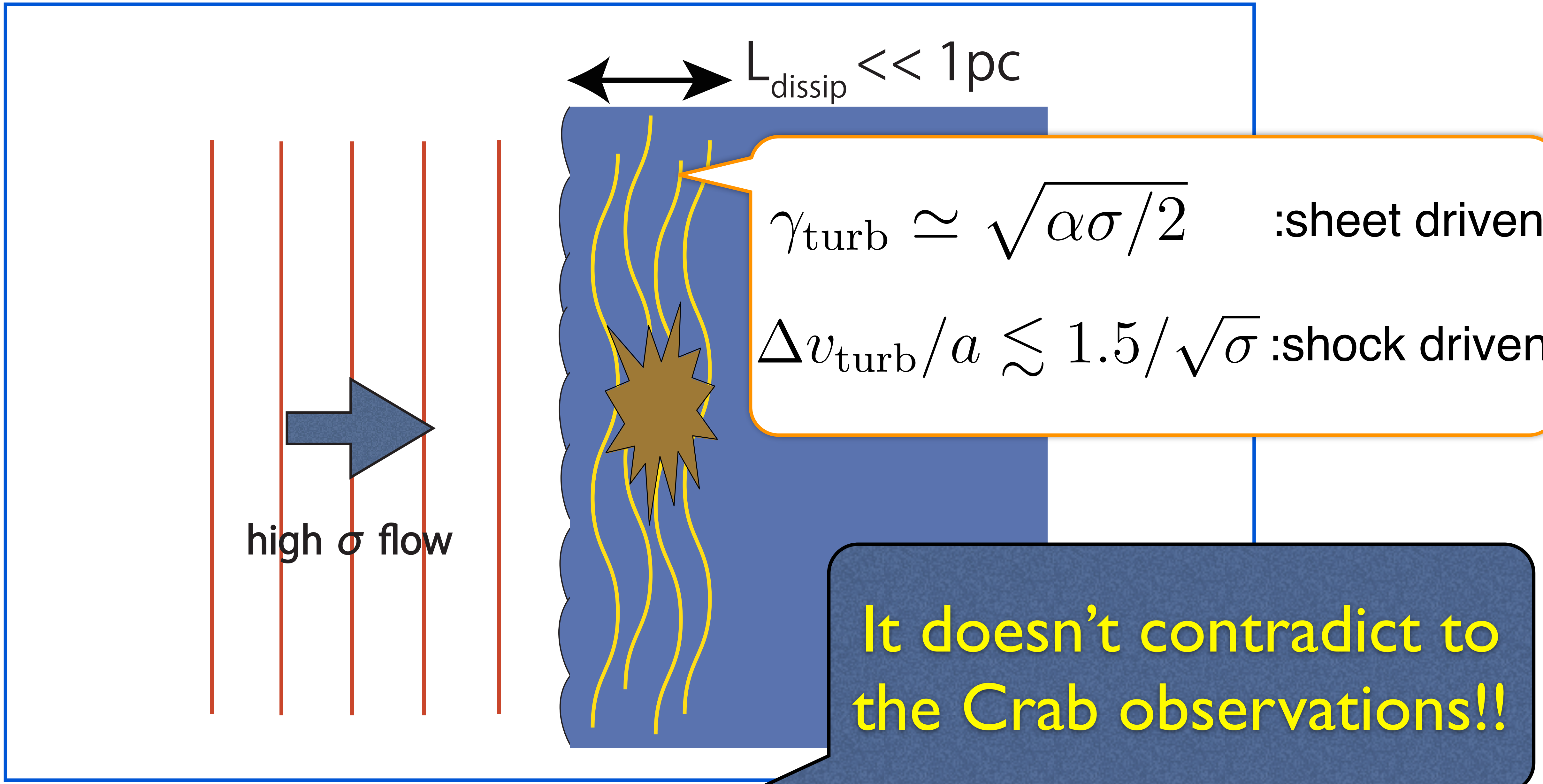
$$-4\epsilon = \nabla \cdot \mathbf{F} + B_0^2 S$$

$$\Rightarrow \quad \epsilon_{\text{eff}} = \epsilon + B_0^2 S/4$$

$$\langle \Psi_{\mathbf{v}} \rangle = \frac{B_0^2}{2} \left\langle \delta(\boldsymbol{\nabla} \cdot \mathbf{v}) \delta\left(\frac{1}{\sqrt{\rho}}\right) \bar{\delta}(\sqrt{\rho}) - \bar{\delta}(\boldsymbol{\nabla} \cdot \mathbf{v}) \right\rangle,$$

$$\langle \Psi_{\mathbf{v_A}} \rangle$$
$$= \mathbf{B_0} \cdot \left\langle \boldsymbol{\nabla}\left(\frac{1}{\sqrt{\rho}}\right)\left[(\mathbf{B_0} \cdot \mathbf{v}')\left\{\rho' \delta\left(\frac{1}{\sqrt{\rho}}\right)\right\} - (\mathbf{B_0} \cdot \mathbf{v}) \frac{\delta\rho}{2\sqrt{\rho'}}\right] - \boldsymbol{\nabla}'\left(\frac{1}{\sqrt{\rho'}}\right)\left[(\mathbf{B_0} \cdot \mathbf{v})\left\{\rho \delta\left(\frac{1}{\sqrt{\rho}}\right)\right\} - (\mathbf{B_0} \cdot \mathbf{v}') \frac{\delta\rho}{2\sqrt{\rho}}\right]\right\rangle.$$

11. Application — Relativistic Jets



$$\frac{l_{\mathrm{dissip,obs}}}{r_{\mathrm{TS}}} \sim 10^{-4} \left(\frac{\sigma}{10^4}\right) \left(\frac{2\pi r_{\mathrm{LC}}}{10^9[\mathrm{cm}]}\right) \left(\frac{M_{\mathrm{R}}}{0.1}\right)^{-1}$$

2. Poynting Dominated Plasma of Astrophysical Phenomena